数学学院本科生 2023-2024 学年第二学期几何学（全英文）期末考试试卷

授课老师：徐彬斌

Instructions:

(涉及的距离都是欧氏距离)

(写在其它位置要标明题号)

$\underline{e}_i = (0, \cdots, 0, \underset{i}{1}, 0, \cdots, 0) \in \mathbb{R}^n$.

1, Let G be the dihedral group of some regular polygon P in $\mathbb{R}^2$,and $m \in \mathbb{N}^*$ be the number of elements in G.

(a)Compute the number of sides of P.

(b)Compute the interior angle of P.

2，Let

$$\underline{x}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \underline{x}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \underline{x}_3 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}.$$

(a)Give two isometries of $\mathbb{R}^2$ sending $\underline{x}_1$ to $\underline{x}_2$.

(b)Is there an isometry sending $\underline{x}_1$ to $\underline{x}_2$ and $\underline{x}_2$ to $\underline{x}_3$ at the same time?

(c)Compute the orthogonal projection of $\underline{x}_3$ to the line $L(\underline{x}_1, \underline{x}_2)$ passing through $\underline{x}_1$ and $\underline{x}_2$.

3, Consider the map

$$f : \{1, 2, \cdots, 8\} \to \{1, 2, \cdots, 8\}$$

defined by

x	1	2	3	4	5	6	7	8
$f(x)$	2	3	4	1	8	6	5	7

Let G be the group generated by f. Consider the G-action on $\{1, 2, \cdots, 8\}$.

(a)Compute the number of G-orbits in $\{1, 2, \cdots, 8\}$.

(b)How many elements of $\{1, 2, \cdots, 8\}$ in each G-orbit?

(c)Compute Stab(4).

4, Let f and g be two affine transformations of $\mathbb{R}^3$. We would like to use the following method to see if $f = g$:

(1)Choose finitely many points $\underline{x}_1, \cdots, \underline{x}_k \in \mathbb{R}^3$.

(2)Check if $f(\underline{x}_1) = g(\underline{x}_1), f(\underline{x}_2) = g(\underline{x}_2), \cdots$ and $f(\underline{x}_k) = g(\underline{x}_k)$. If yes, then $f = g$; if no, then $f \neq g$.

In order to make this method work, at least how many points do we need? What is the minimal possible value of k?

5, Consider the group $\operatorname{Aff}\mathbb{R}^3$ and its action on lines in $\mathbb{R}^3$.

(a)Compute $\operatorname{Stab}(L(\underline{0}, \underline{e}_1))$.

(b)Compute $\operatorname{Stab}(L(\underline{0}, \underline{e}_1)) \cap \operatorname{Stab}(L(\underline{0}, \underline{e}_2))$.

6, Consider lines in $\mathbb{R}^2$ passing the origin. Take the affine chart

$$\mathbb{A} = \left\{ \begin{bmatrix} 1 \\ x_2 \end{bmatrix} \middle| x_2 \in \mathbb{R} \right\},$$

and call the corresponding x_2 of a line passing the origin its affine coordinate.

Denote by L_1, L_2, L_3, L_4 the lines with coordinates $1, 2, 3, 4$ respectively, and L_1', L_2', L_3', L_4' the lines with coordinates $-1, -3, -2, -4$ respectively.

(a)Is there a projective transformation sending (L_1, L_2, L_3, L_4) to (L_1', L_2', L_3', L_4')? If yes, how many such projective transformations are there?

(b)Is there a projective transformation sending $\{L_1, L_2, L_3, L_4\}$ to $\{L_1', L_2', L_3', L_4'\}$? If yes, how many such projective transformations are there?

（回忆者的注：“sending (L_1, L_2, L_3, L_4) to (L_1', L_2', L_3', L_4')” 表示把 L_1 送到 L_1', L_2 送到 L_2', L_3 送到 L_3', L_4 送到 L_4', 顺序要对应; 而“sending $\{L_1, L_2, L_3, L_4\}$ to $\{L_1', L_2', L_3', L_4'\}$”表示把集合 $\{L_1, L_2, L_3, L_4\}$ 送到集合 $\{L_1', L_2', L_3', L_4'\}$，$L_1$ 不一定要去 L_1', 也可以去 L_2', L_3'或L_4'. 考试时老师简单解释了这两个符号的区别.）

7, Pascal's theorem: consider the following hexagon inscribed in a circle. Denote by $a, \cdots, f$ its vertices, and let

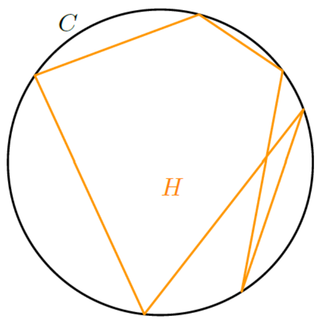
$$p = L(a, b) \cap L(d, e),$$
$$q = L(b, c) \cap L(e, f),$$
$$r = L(c, d) \cap L(f, a).$$

Pascal's theorem tells us that p, q, r are collinear.

(a)Mark the vertices $a, \cdots, f$ and the intersections p, q, r in the picture.

(b)Draw a new picture to show the dual theorem to Pascal's theorem. What does the dual theorem tell us about?

（回忆者的注：1. 我忘了卷子有没有给出 Pascal 定理的结论；2. 卷子上的图我没画，大概参考下面这个吧.）


C
H